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VI Semester B.A./B.Sc. Examination, May/June - 2019

MATHEMATICS

Mathematics - VII

(CBCS) (F+R) (2016-17 & Onwards)

Time : 3 Hours Max. Marks : 70

Instructions : *Answer **all** questions.*

PART - A

Answer **any five** sub-questions. **5x2=10**

1. (a) In a vectorspace V(F) show that $C(-\alpha)=-(C\alpha),\ \forall C\in F,\ \alpha\in V$

(b) Prove that the set $S=\{(3, 2, -1), (0, 4, 5), (6, 4, -2)\}$ is Linearly dependent in $V_3(R)$.

(c) Find the matrix of the linear transformation

$T: V_2(R) \to V_2(R)$ defined by

$T(x, y)=(2x+3y, 4x-5y)$ with respect to standard bases.

(d) Define Rank and Nullity of linear transformation.

(e) In a cylindrical coordinate system prove that $\hat{e}_\phi \cdot \hat{e}_z = 0$

(f) Solve $\dfrac{x\,dx}{y^2 z} = \dfrac{dy}{zx} = \dfrac{dz}{y^2}$

(g) Form the partial differential equation by eliminating the arbitrary constants from $2Z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$

(h) Solve $\sqrt{p} + \sqrt{q} = 1$

P.T.O.

PART - B

2x10=20

Answer **any two full** questions.

2. (a) A Subset W of a vectorspace V(F) is a subspace if and only if
   (i) $\alpha, \beta \in W \Rightarrow \alpha + \beta \in W$
   (ii) $C \in F, \alpha \in W \Rightarrow C \cdot \alpha \in W$
   (b) Find the basis and dimension of the subspace spanned by (2, 4, 2), (1, −1, 0), (1, 2, 1), (0, 3, 1) in $V_3(R)$

OR

3. (a) A set of non zero vectors $(\alpha_1, \alpha_2, \ldots\ldots \alpha_n)$ of vectorspace V(F) is linearly dependent if and only if one of these vectors say α_k $(2 \le k \le n)$ is expressed as a linear combination of its preceding ones.
   (b) Show that the subset $W = \{(x_1, x_2, x_3) / x_1 + x_2 + x_3 = 0\}$ is a subspace of $V_3(R)$

4. (a) If $T : U \to V$ is a linear transformation then prove that.
   (i) $T(0) = 0'$, where 0 and 0′ are the zero vectors of U and V respectively.
   (ii) $T(-\alpha) = -T(\alpha), \ \forall \alpha \in U$
   (b) Verify whether $T : V_2(R) \to V_2(R)$ is a linear transformation defined by $T(x, y) = (3x + 2y, 3x - 4y)$

OR

5. (a) Find the range space, null space, rank, nullity and hence verify rank nullity theorem for $T : V_3(R) \to V_3(R)$ given by $T(x, y, z) = (x + y, x - y, 2x + z)$
   (b) Show that the linear transformation $T : R^3 \to R^3$ given by $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 + e_3$, $T(e_3) = e_1 + e_2 + e_3$ is non-singular where $\{e_1, e_2, e_3\}$ is the standard basis of R^3.

PART - C

2x10=20

Answer **any two full** questions.

6. (a) Verify the condition for integrability and solve $(2x^2 + 2xy + 2xz^2 + 1)\ dx + dy + 2z\ dz = 0$
   (b) Solve $p \tan x + q \tan y = \tan z$

OR

7. (a) Show that the cylindrical coordinate system is Orthogonal Curvilinear Coordinate System.
   (b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates and find f_ρ, f_ϕ, f_z

8. (a) Solve $\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2 - y^2)}$

(b) Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$

OR

9. (a) Express the vector $\vec{f} = 3x\hat{i} - 2yz\hat{j} + x^2 z\hat{k}$ in cylindrical coordinates and find f_ρ, f_ϕ, f_z

(b) Express the vector $\vec{f} = x\hat{i} - y\hat{j} + z\hat{k}$ in spherical coordinates and find f_r, f_θ, f_ϕ

PART - D

Answer **any two full** questions. **2x10=20**

10. (a) Form the partial differential equation by eliminating the arbitrary functions $z = f(x+ay) + g(x-ay)$

(b) Solve $p(1+q) = zq$

OR

11. (a) Solve $[D^2 - 2DD' + (D')^2]z = e^{x+2y}$

(b) Solve $p + q = \sin x + \sin y$

12. (a) Find the complete integral of $px + qy = pq$ by Charpit's method

(b) Solve $[D^2 - 2DD' + (D')^2]z = 12\,xy$

OR

13. (a) A tightly streched string with fixed end points $x=0$ and $x=1$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{1}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$.

(b) Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ subjected to the conditions.

(i) $u(0, t) = 0,\ u(1, t) = 0$ for all t

(ii) $u(x, 0) = x^2 - x,\ 0 \le x \le 1$

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